Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 1 (Individual)

香港数学竞赛 (1998 – 99)

决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若一个 P 边的多边形的角形成一算术级数，且最小和最大的角分别为 $20°$ 及 $160°$，求 P 之值。

If the angles of a P-sided polygon form an Arithmetic Progression and the smallest and the largest angles are $20°$ and $160°$ respectively , find the value of P .

$P =$

(ii) 在 ΔABC 中，$AB = 5$，$AC = 6$ 及 $BC = P$。若 $\frac{1}{Q} = \cos 2A$，求 Q 之值。

(提示： $\cos 2A = 2\cos^2 A - 1$)

In ΔABC , $AB = 5$, $AC = 6$ and $BC = P$. If $\frac{1}{Q} = \cos 2A$, find the value of Q .

(Hints : $\cos 2A = 2\cos^2 A - 1$)

$Q =$

(iii) 若 $\log_2 Q + \log_4 Q + \log_8 Q = \frac{R}{2}$，求 R 之值。

If $\log_2 Q + \log_4 Q + \log_8 Q = \frac{R}{2}$, find the value of R .

$R =$

(iv) 若两数 R 和 $\frac{11}{S}$ 的积等于它们的和，求 S 之值。

If the product of the numbers R and $\frac{11}{S}$ is the same as their sum , find the value of S .

$S =$

Hong Kong Mathematics Olympiad (1998 – 99)
Final Event 2 (Individual)
香港数学竞赛 (1998 – 99)
决赛项目 2 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若 x、y、z 为非零实数使得 $\frac{x+y-z}{z}=\frac{x-y+z}{y}=\frac{-x+y+z}{x}$，且 $a=\frac{(x+y)(y+z)(z+x)}{xyz}$，求 a 之值。

If x, y, z are non-zero real numbers such that $\frac{x+y-z}{z}=\frac{x-y+z}{y}=\frac{-x+y+z}{x}$ and $a=\frac{(x+y)(y+z)(z+x)}{xyz}$, find the value of a.

$a =$

(ii) 设 u 和 t 为正整数使得 $u+t+ut=4a+2$。若 $b=u+t$，求 b 之值。

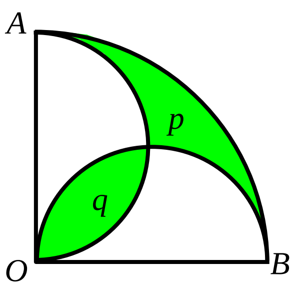
Let u and t be positive integers such that $u+t+ut=4a+2$. If $b=u+t$, find the value of b.

$b =$

(iii) 在图一，OAB 为四分之一圆，且以 OA、OB 为直径绘出两个半圆。若 p、q 代表斜线部份之面积，其中 $p=(b-9)\ \text{cm}^2$ 及 $q=c\ \text{cm}^2$，求 c 之值。

In Figure 1, OAB is a quadrant of a circle and semi-circles are drawn on OA and OB. If p, q denotes the areas of the shaded regions, where $p=(b-9)\ \text{cm}^2$ and $q=c\ \text{cm}^2$, find the value of c.

$c =$



图一
Figure 1

(iv) 设 $f_0(x)=\dfrac{1}{c-x}$，且 $f_n(x)=f_0\left(f_{n-1}(x)\right)$，$n = 1, 2, 3, \ldots$。若 $f_{2000}(2000)=d$，求 d 之值。

Let $f_0(x)=\dfrac{1}{c-x}$, and $f_n(x)=f_0\left(f_{n-1}(x)\right)$, $n=1, 2, 3, \ldots$. If $f_{2000}(2000)=d$, find the value of d.

$d =$

Hong Kong Mathematics Olympiad (1998 – 99)
Final Event 3 (Individual)
香港数学竞赛 (1998 – 99)
决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 对任意整数 m 及 n，$m\otimes n$ 之定义如下：$m\otimes n=m^n+n^m$。若 $2\otimes a=100$，求 a 之值。

For all integers m and n, $m\otimes n$ is defined as: $m\otimes n=m^n+n^m$. If $2\otimes a=100$, find the value of a.

$a=$

(ii) 若 $\sqrt[3]{13b+6a+1}-\sqrt[3]{13b-6a-1}=\sqrt[3]{2}$，其中 $b>0$，求 b 之值。

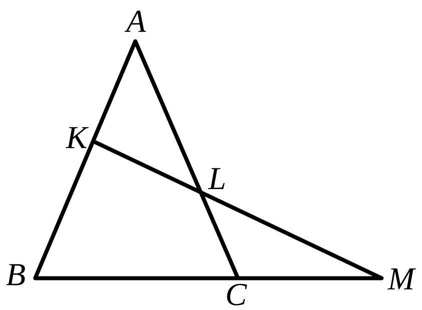
If $\sqrt[3]{13b+6a+1}-\sqrt[3]{13b-6a-1}=\sqrt[3]{2}$, where $b>0$, find the value of b.

$b=$

(iii) 在图二，$AB=AC$ 和 $KL=LM$。若 $LC=b-6$ cm 及 $KB=c$ cm，求 c 之值。

In Figure 2, $AB=AC$ and $KL=LM$. If $LC=b-6$ cm and $KB=c$ cm, find the value of c.

$c=$



图二
Figure 2

(iv) 数列 $\{a_n\}$ 的定义如下: $a_1 = c$, $a_{n+1} = a_n + 2n$ $(n \geq 1)$。若 $a_{100} = d$, 求 d 之值。

The sequence $\{a_n\}$ is defined as : $a_1 = c$, $a_{n+1} = a_n + 2n$ $(n \geq 1)$. If $a_{100} = d$, find the value of d.

$d =$

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 4 (Individual)

香港数学竞赛 (1998 – 99)

决赛项目 4 (个人)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 李先生今年 a 岁。若把李先生出生的月份与 a 相乘，其结果是 253，求 a 的值。

Mr. Lee is a years old. If the product of a and his month of birth is 253, find the value of a.

$a =$

(ii) 李先生有糖 $a+b$ 粒。若平均分给 10 人，则余下 5 粒。若平均分给 7 人，则欠 3 粒。求 b 之最小值。

Mr. Lee has $a+b$ sweets. If he divides them equally among 10 persons, 5 sweets will be remained. If he divides them equally among 7 persons, 3 more sweets are needed. Find the minimum value of b.

$b =$

(iii) 设 c 为一正实数。若 $x^2+2\sqrt{c}x+b=0$ 仅有一实数解，求 c 之值。

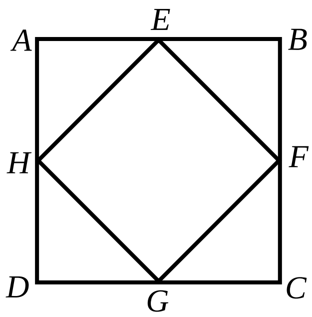
Let c be a positive real number. If $x^2+2\sqrt{c}x+b=0$ has one real root only, find the value of c.

$c =$

(iv) 在图三，正方形 $ABCD$ 之面积为 d。若 E 、F 、G 、H 分别是 AB 、BC 、CD 、 DA 之中心点，且 $EF=c$， 求 d 之值。

In Figure 3 , the area of the square $ABCD$ is equal to d . If E , F , G , H are the mid-points of AB , BC , CD and DA respectively , and $EF=c$, find the value of d .

$d=$



图三

Figure 3

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 5 (Individual)

香港数学竞赛 (1998 – 99)

决赛项目 5 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 若 $144^p = 10$、$1728^q = 5$ 及 $a = 12^{2p-3q}$，求 a 之值。

If $144^p = 10$, $1728^q = 5$ and $a = 12^{2p-3q}$, find the value of a.

$a =$

(ii) 若 $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ 及 $\frac{a}{x} = b$，求 b 之值。

If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ and $\frac{a}{x} = b$, find the value of b.

$b =$

(iii) 若方程式 $x^2 - bx + 1 = 0$ 有 c 个实数解，求 c 之值。

If the number of real roots of equation $x^2 - bx + 1 = 0$ is c, find the value of c.

$c =$

(iv) 设 $f(1) = c + 1$ 及 $f(n) = (n-1)f(n-1)$，其中 $n > 1$。若 $d = f(4)$，求 d 之值。

Let $f(1) = c + 1$ and $f(n) = (n-1)f(n-1)$, where $n > 1$. If $d = f(4)$, find the value of d.

$d =$